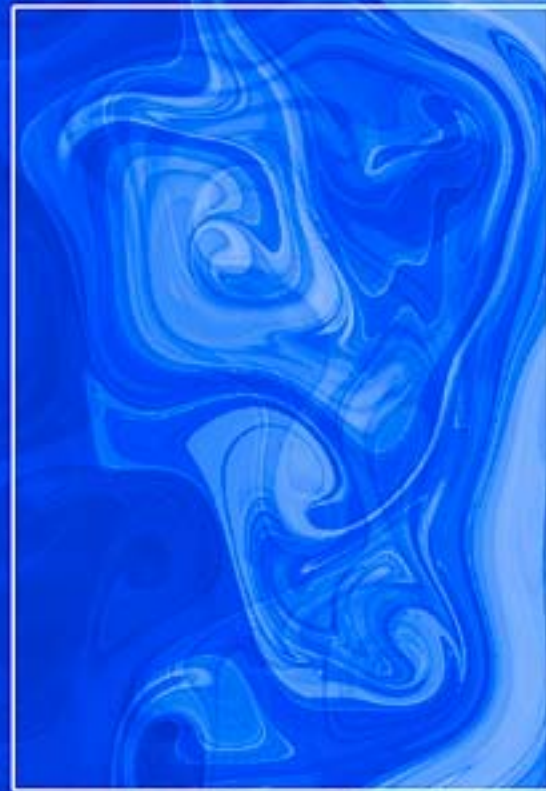


## BEAUTIFUL GEOMETRIES UNDERLYING OCEAN NONLINEAR PROCESSES

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*Nonlinear Processes in Geophysics (NPG)*



**Finding order in the apparent chaos** that seems to govern ocean motions is a formidable task which has drawn the attention of scientists and oceanographers all over the world for the last decades. The endeavour of describing how heat, salt, carbon dioxide and other biogeochemical tracers are transported in the ocean has become a global challenge, and its understanding is of vital importance for predicting and assessing their impact on global climate change or the distribution of natural marine resources.

The main agents that drive ocean circulation at different scales are large-scale ocean currents such as the Gulf Stream, Kuroshio, Antarctic Circumpolar Current; mesoscale eddy structures such as the Agulhas rings, or sub-mesoscale eddies found near coasts. Nowadays, these structures can be directly observed by satellite altimeters, high-frequency coastal radars or through in situ devices. While in the past, navigators such as Juan Ponce de León noticed their presence on expeditions.

While strong currents are thought to be responsible for most of the major transport processes, eddies determine much of the mixing in the ocean. However, a common aspect of both features is that they define regions where fluid parcels have difficulty crossing, which are known in the literature as transport barriers. These water barriers form boundaries between fluids with different physical properties and give rise to an organizational structure in the overall picture of ocean flows. Eddy-induced transport has been historically underestimated, but increasingly it is being recognized as a key contributor to ocean transport processes. Indeed, since vortices are robust, long-lived structures that may persist for periods lasting from months to years, when water gets eventually trapped within the eddy's core, the fluid inside will travel hundreds to thousands of kilometres within the mesoscale structure and preserve its biogeochemical properties for a long time.

The ocean motion follows a fully nonlinear dynamics in which turbulence is an essential feature, enabling interactions between motions on different spatial scales and playing a key role in ocean transport and mixing. Efforts to understand and parameterize turbulent mixing have been a research focus for many years, and continue to be essential towards understanding and predicting the evolution of the Earth's oceans.

In this context, dynamical systems theory has provided a framework for describing messy paths of float trajectories in ocean flows. Behind their apparent disorder, a subtle and sophisticated order is revealed through geometrical structures over many scales. The underlying fabric uncovered by dynamical systems tools has been considered as the skeleton of turbulence. These structures, which organize trajectories schematically into distinct regions corresponding to qualitatively different types of trajectories, are responsible for the transport and mixing processes that govern the ocean in these areas. The boundaries, or barriers, between these regions are mathematically realized as objects called manifolds. The figure on the left makes visible this geometrical representation for the Gulf Stream on the 14 March 2004 over an area extending 1100 km in length. The image is produced using altimeter data sets distributed by AVISO (<http://www.aviso.oceanobs.com/duacs/>). This representation is achieved using Lagrangian Descriptors, a mathematical tool which highlights invariant manifolds in flows with a general time dependence.

A magnification of the area indicated by the rectangle is shown on the next page right. An intricate geometrical pattern, a chaotic tangle, is observed at a lower scale, in which the filamentary structures are less than one kilometre in length. The skeleton displayed in these figures represents time dependent dynamical barriers along which passive ocean tracers are attracted either backwards or forwards in time.

The figures summarize particle histories over a time interval of 140 days: 70 days after the 14 March 2004 and 70 days before this date. Should this time interval be reduced then the complexity of the figure is also reduced.

The six small figures show a sequence of images summarizing particle histories. They show a small region of the Gulf Stream with a different colour scale and forward and backward time intervals of 15, 30 and 50 days, respectively. Filamentary patterns are related to the mixing of passive scalars during the time interval over which the figures are calculated. The bottom panels show the forward evolution of two particle sets, red and blue, according to the Aref

Geometrical representation for the Gulf Stream on 14 March 2004 as revealed by the Lagrangian Descriptor function. The image covers an area of 1100 by 600 km.

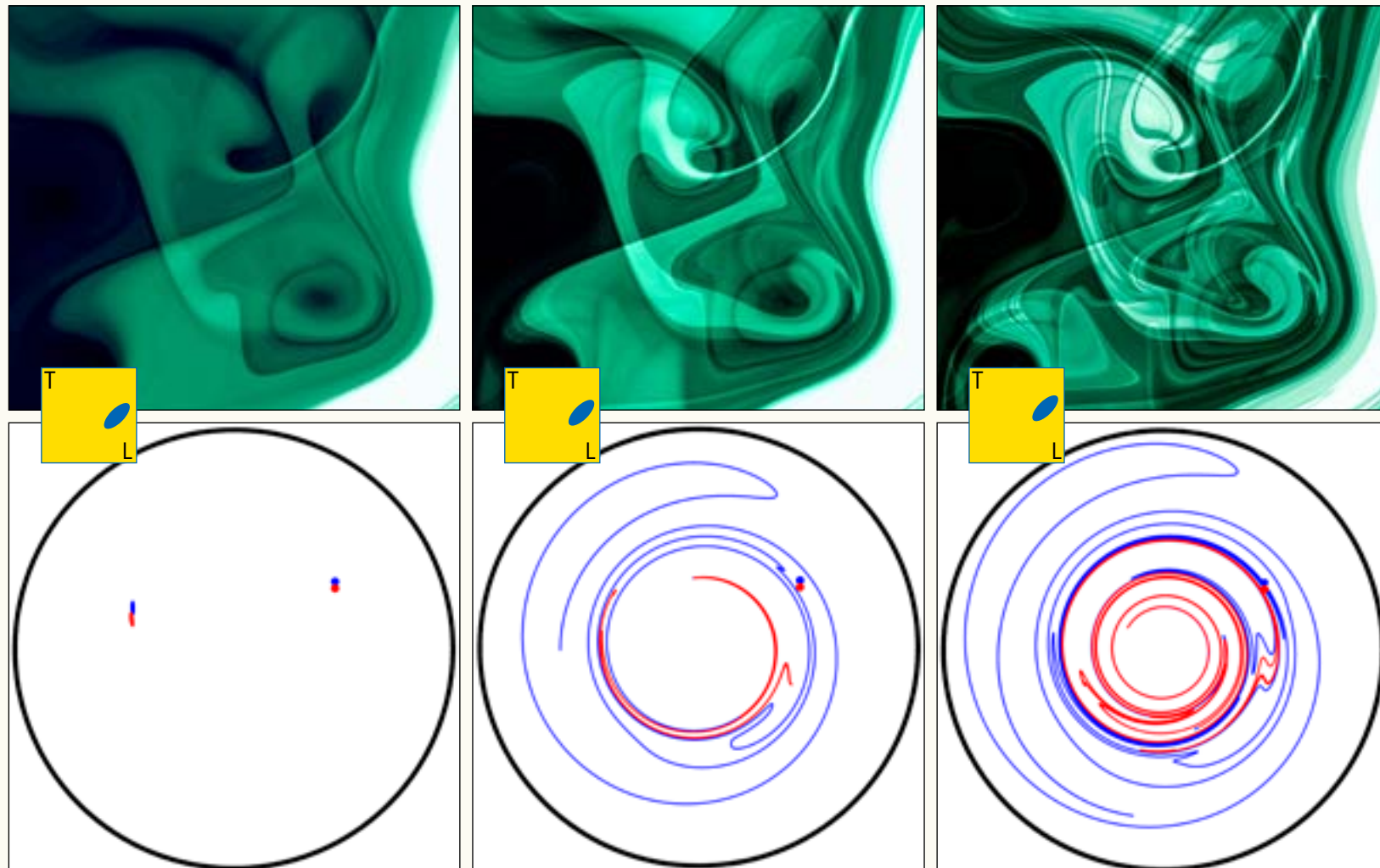


Magnification of Gulf Stream from previous page (270 by 150 km)

*Below*  
Sequence of images summarizing particle histories  
in the Gulf Stream with time intervals of 15, 30 and  
50 days (top) and Aref blinking vortex maps (bottom)

blinking vortex map (a conceptualisation of flow patterns related to a pair of vortices, that switch on and off alternately). After a short flow period, particles are not mixed, their evolution does not show complicated patterns. However, at longer times this structure becomes more and more labyrinthine, showing that attracting filaments and particles are intermingled.

The structures shown in the figures are more than just beautiful mathematical objects obtained from the nonlinear ocean motion: they are objects truly present in nature. The image of sea ice off the east coast of Greenland on the following pages shows structures strikingly similar to those displayed in the analysis figures. The swirling structures marked out by the sea ice are visualizations of the unstable manifolds



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[http://youtu.be/P-f4k-cjk\\_8](http://youtu.be/P-f4k-cjk_8)



Sea ice off the east coast of Greenland taken by NASA's Aqua satellite on 17 October 2012.